Simulation of unsteady fluid flow and heat transfer by BEM

Jure Ravnik and Leopold Škerget

jure.ravnik@uni-mb.si, http://iepoi.uni-mb.si/ravnik/research.html

University of Maribor, Faculty of Mechanical Engineering,

Smetanova 17, 2000 Maribor, Slovenia

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$\vec{v} - \vec{\omega}$ *formulation of N-S*

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- The velocity-vorticity formulation of Navier-Stokes equations consists of the kinematics equation and the vorticity transport equation.
- The kinematics equation is a vector elliptic partial differential equation of Poisson type and links the velocity and vorticity fields for every point in space and time. For an incompressible fluid it can be stated as

$$\nabla^2 \vec{v} + \vec{\nabla} \times \vec{\omega} = 0,$$

  where we must bear in mind, that both velocity and vorticity fields are divergence free.

$\vec{v} - \vec{\omega}$ formulation of N-S

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velocity

$\vec{v} - \vec{\omega}$ formulation of N-S

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vorticity; $\vec{\omega} = \vec{\nabla} \times \vec{v}$

$\vec{v} - \vec{\omega}$ formulation of N-S

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- The kinetic aspect of fluid movement is governed by the vorticity transport equation. Buoyancy is modelled within the Boussinesq approximation. Density variations with temperature $\rho(T) = \rho_0[1 - \beta_T(T - T_0])$ are considered only in the buoyancy term and defined by the thermal volume expansion coefficient β_T and the temperature difference. Using these assumptions we may write the vorticity transport equation as:

$$\frac{\partial \vec{\omega}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{\omega} = (\vec{\omega} \cdot \vec{\nabla})\vec{v} + \frac{1}{Re}\nabla^2\vec{\omega} - \frac{Ra}{PrRe^2}\vec{\nabla} \times T\vec{g}$$

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Reynolds number; $Re = \frac{v_0 \mathcal{L}}{\nu}$

$\vec{v} - \vec{\omega}$ formulation of N-S

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Rayleigh number; $Ra = \frac{g_0 \beta_T \Delta T \mathcal{W}^3}{\nu_0 \alpha_0}$

$\vec{v} - \vec{\omega}$ formulation of N-S

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Prandtl number; $Pr = \nu_0/\alpha_0$

$\vec{v} - \vec{\omega}$ formulation of N-S

- We further assume that no internal energy sources are present in the fluid. We will not deal with high velocity flow of highly viscous fluid, hence we will neglect irreversible viscous dissipation. With this, the internal energy conservation law, written with temperature as the unknown variable, reads as:

$$\frac{\partial T}{\partial t} + (\vec{v} \cdot \vec{\nabla})T = \frac{1}{RePr}\nabla^2 T.$$

Integral form of equations

- The integral form of the kinematics equation reads:

$$c(\vec{\theta})\vec{v}(\vec{\theta}) + \int_\Gamma \vec{v}\,\vec{\nabla}u^\star \cdot \vec{n}d\Gamma = \int_\Gamma \vec{v} \times (\vec{n} \times \vec{\nabla})u^\star d\Gamma + \int_\Omega (\vec{\omega} \times \vec{\nabla}u^\star)d\Omega.$$

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$\vec{\theta}$ is the source or collocation point

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the fundamental solution for the diffusion operator; $u^\star = \frac{1}{4\pi|\vec{\theta}-\vec{r}|}$.

Integral form of equations

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- The integral form of the steady vorticity transport equation is:

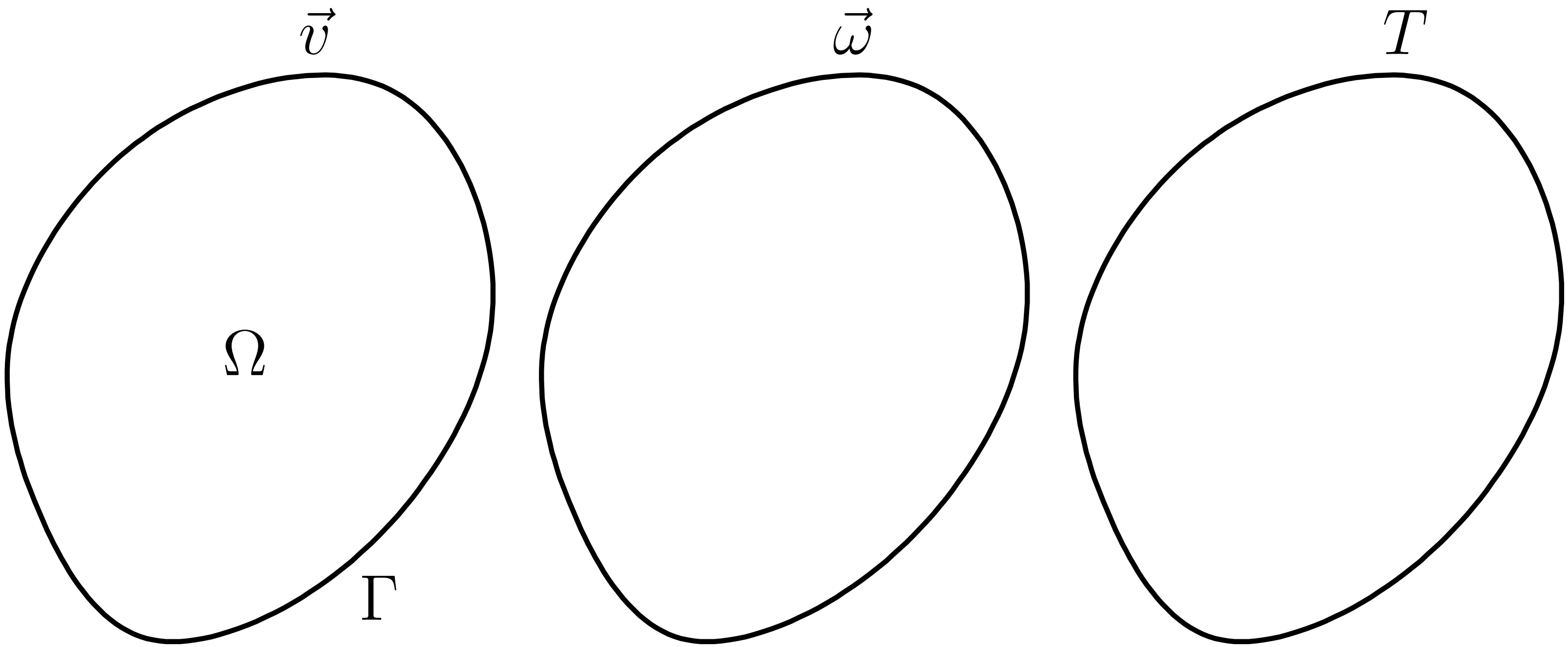
$$c(\vec{\theta})\vec{\omega}(\vec{\theta})+\int_\Gamma \vec{\omega}\vec{\nabla}u^\star\cdot\vec{n}d\Gamma = \int_\Gamma u^\star\vec{q}d\Gamma + Re\int_\Omega u^\star\left\{(\vec{v}\cdot\vec{\nabla})\vec{\omega} - (\vec{\omega}\cdot\vec{\nabla})\vec{v}\right\}d\Omega$$

$$+\frac{Ra}{RePr}\int_\Omega u^\star\vec{\nabla}\times T\vec{g}d\Omega,$$

- The integral form of the steady energy equation is:

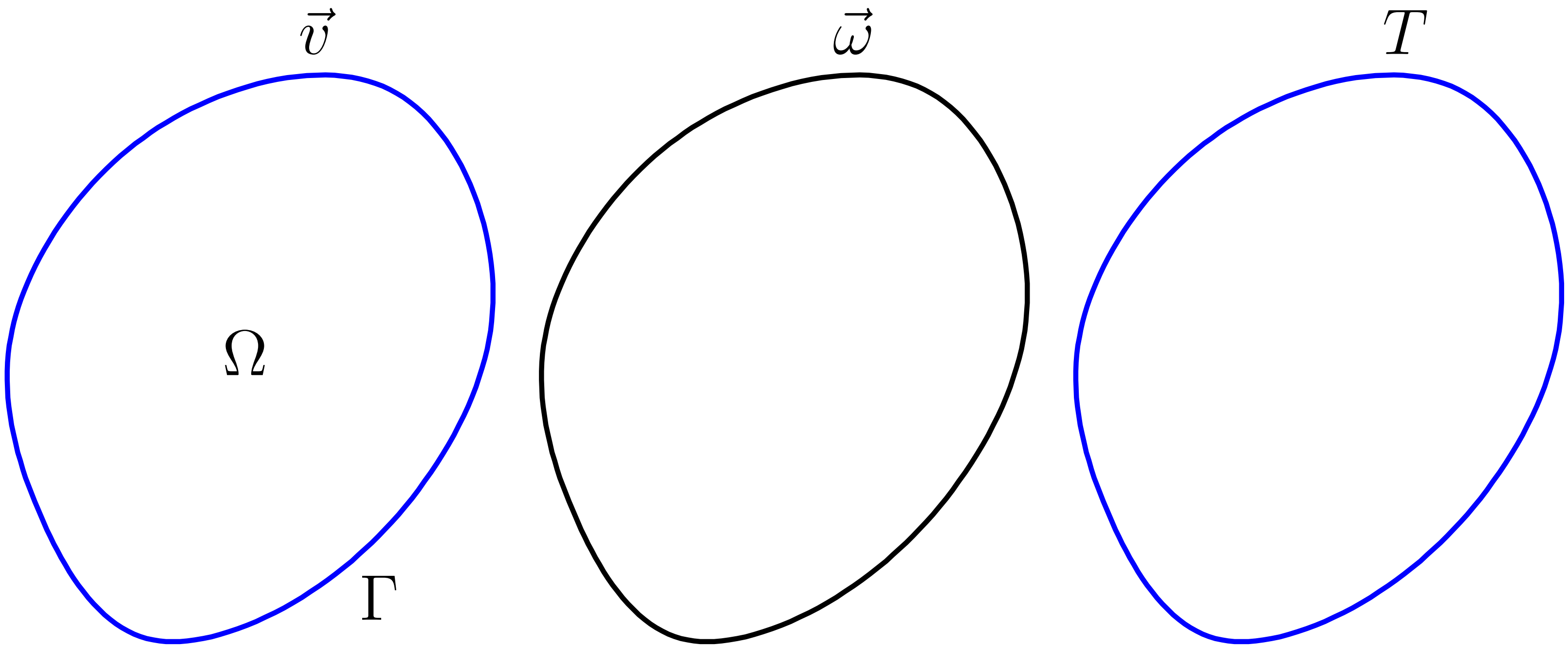
$$c(\vec{\theta})T(\vec{\theta}) + \int_\Gamma T\vec{\nabla}u^\star\cdot\vec{n}d\Gamma = \int_\Gamma u^\star\vec{q}_T d\Gamma + RePr\int_\Omega u^\star(\vec{v}\cdot\vec{\nabla})Td\Omega$$

Numerical Algorithm



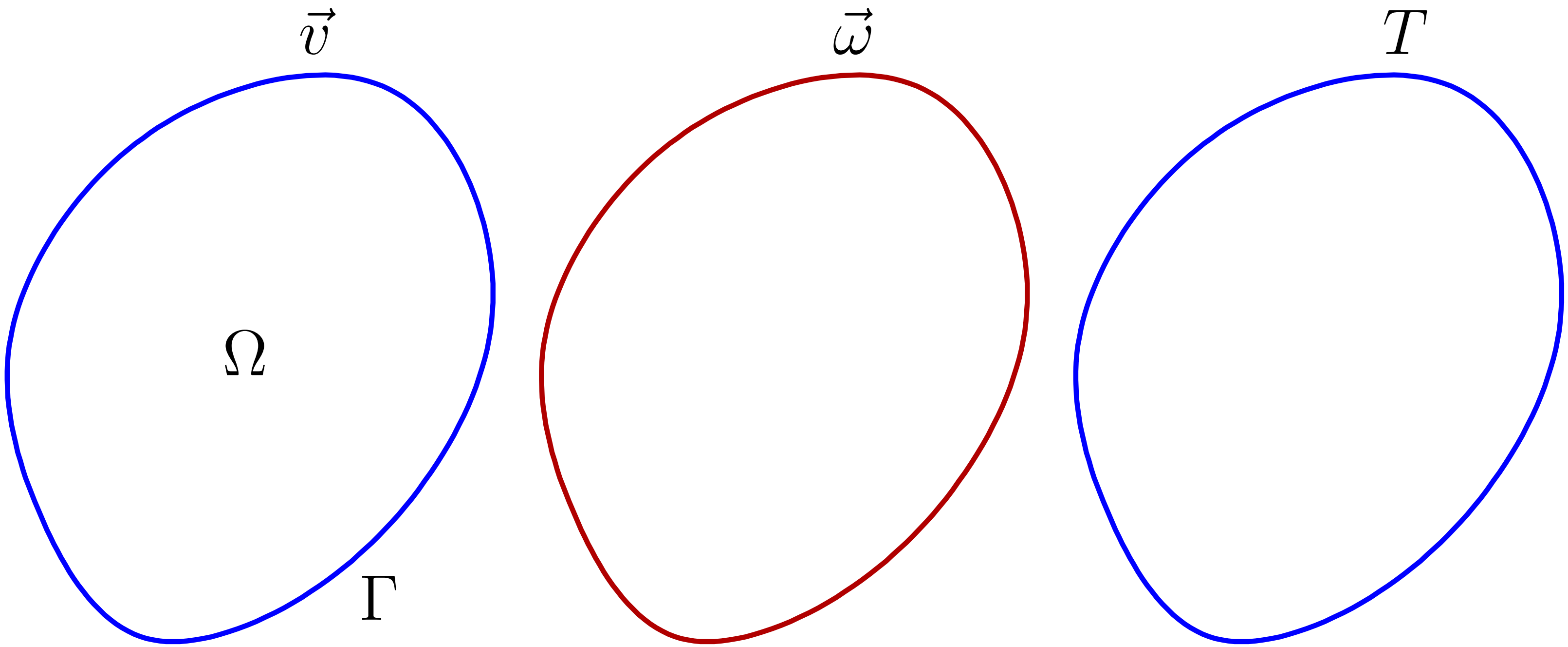
- Considering a domain Ω with boundary Γ we depict the velocity $\vec{v}$, vorticity $\vec{\omega}$ and temperature T fields.

Numerical Algorithm



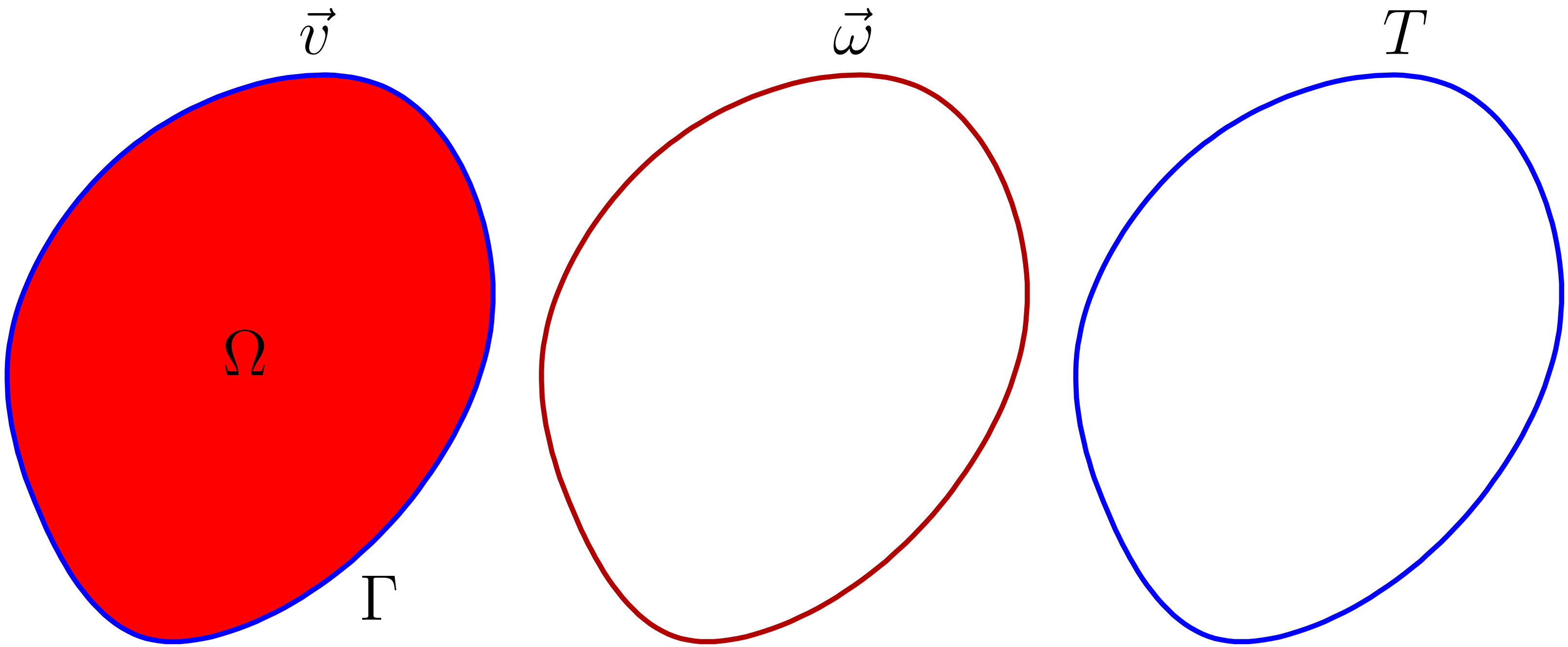
- The boundary conditions known are Dirichlet or Neumann type for velocity and temperature fields. Boundary vorticity values are unknown.

Numerical Algorithm



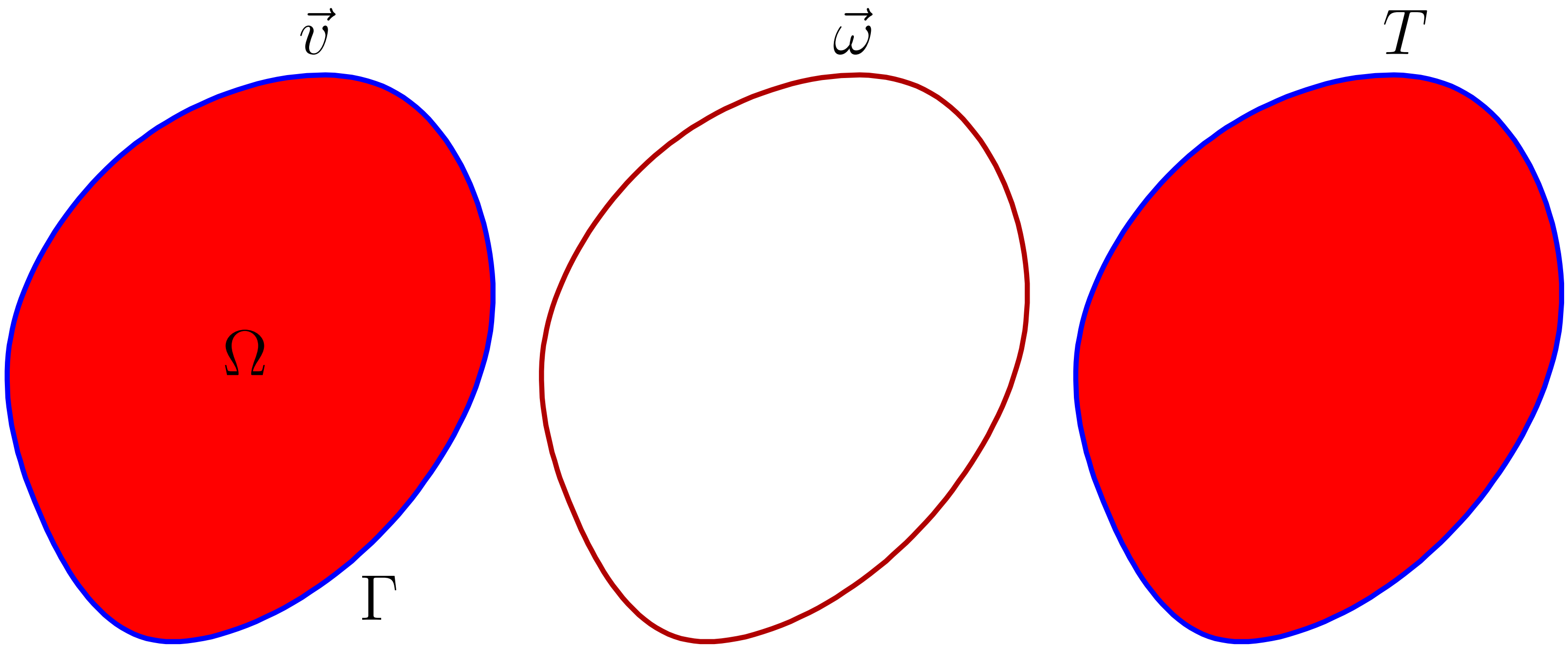
- In each iteration of the nonlinear loop we firstly calculate boundary vorticity values by solving the kinematics equation using single-domain BEM approach

Numerical Algorithm



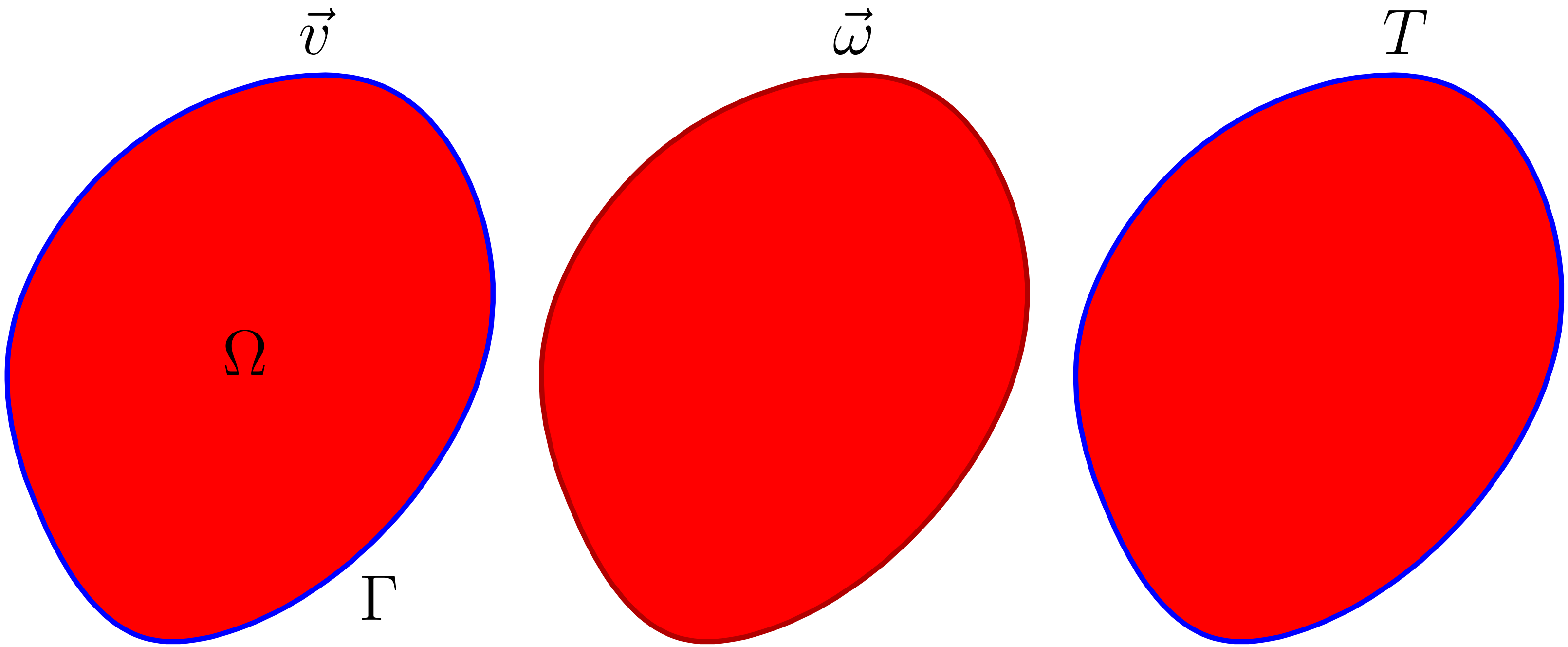
- The kinematic equation is solved again by sub-domain BEM for domain velocities taking into account the newly calculated boundary vorticity values

Numerical Algorithm



- The new velocity field is used to calculate domain temperature values by solving the energy equation by sub-domain BEM.

Numerical Algorithm



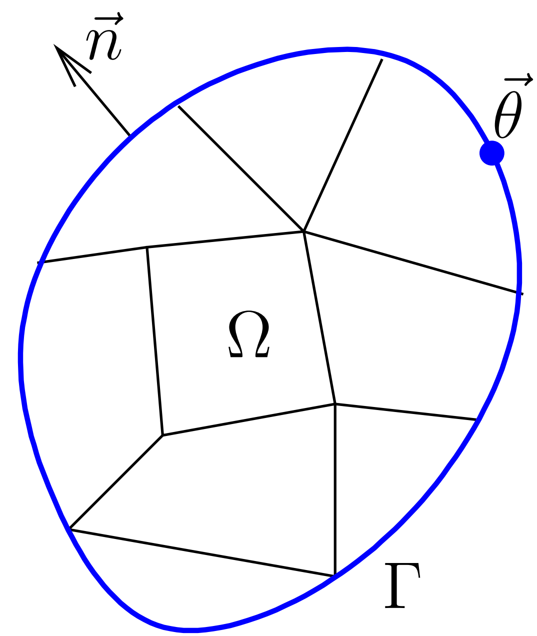
- Finally the new domain vorticity values are obtained by solving the vorticity transport equation using sub-domain BEM.

Single domain BEM

The integral form of the kinematics equation without derivatives of the velocity and vorticity fields takes the following form:

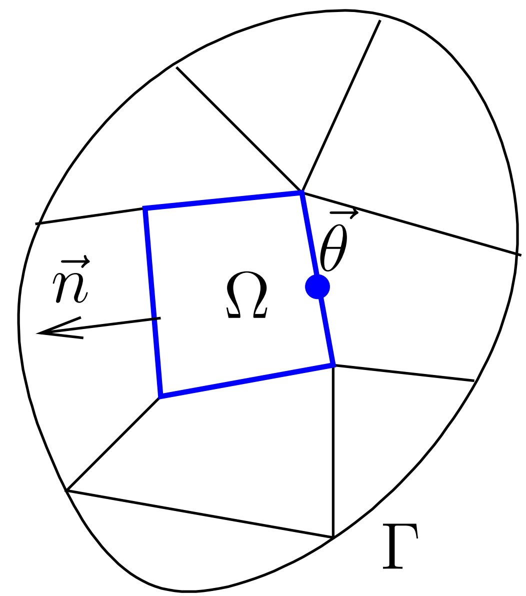
$$c(\vec{\theta})\vec{v}(\vec{\theta}) + \int_\Gamma \vec{v}\,\vec{\nabla}u^\star \cdot \vec{n} d\Gamma = \int_\Gamma \vec{v} \times (\vec{n} \times \vec{\nabla})u^\star d\Gamma + \int_\Omega (\vec{\omega} \times \vec{\nabla}u^\star) d\Omega.$$

- The source (collocation) point $\vec{\theta}$ is set to all nodes on the outer boundary Γ.
- The resulting linear system of equations is governed by fully populated matrices.

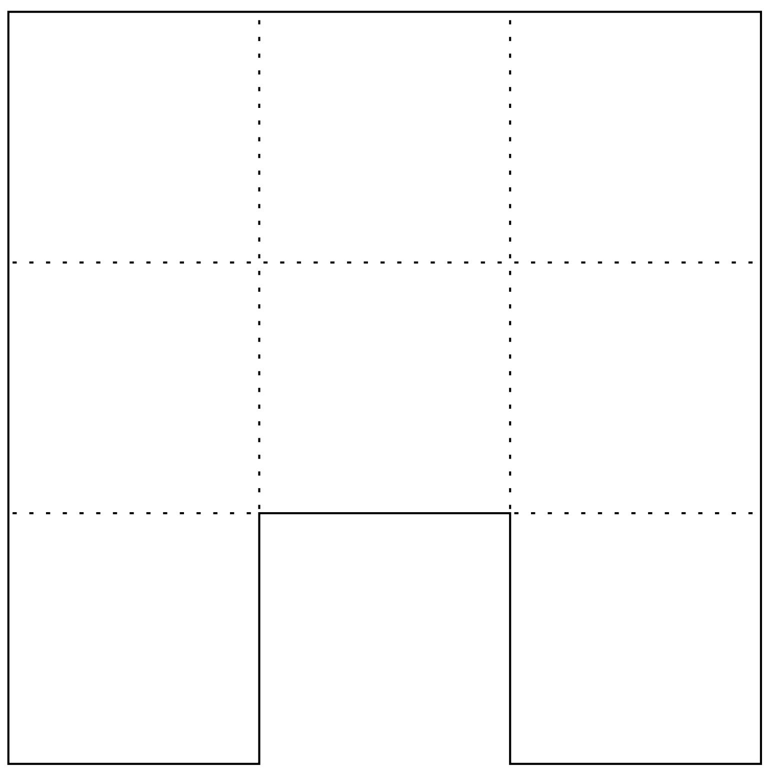


Sub-domain BEM

- The source (collocation) point $\vec{\theta}$ is set to all nodes of each sub-domain.
- Compatibility conditions between subdomains are required.
- The resulting linear system of equations is over-determined and governed by sparsely populated matrices.

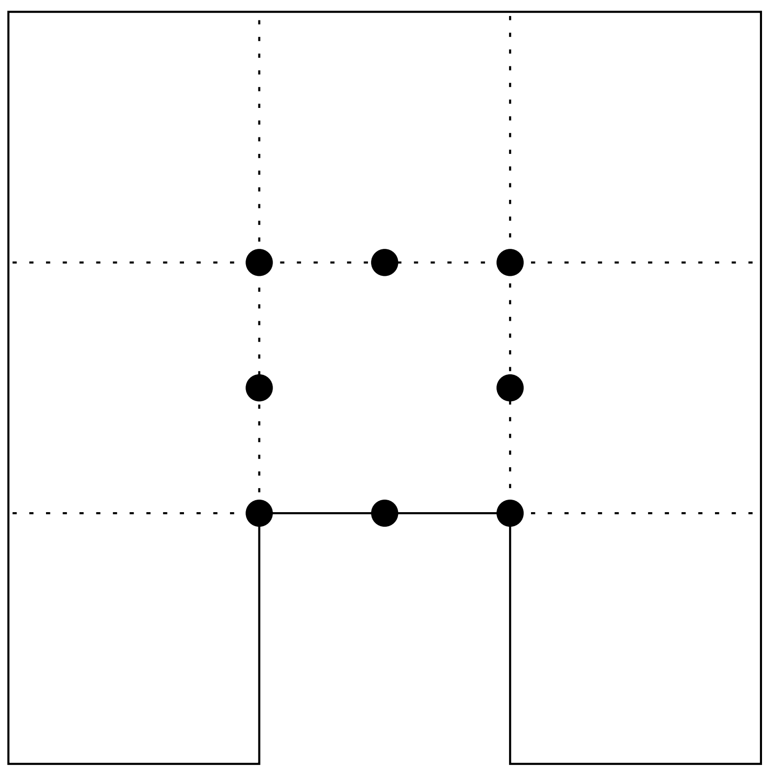


Setting up SLE



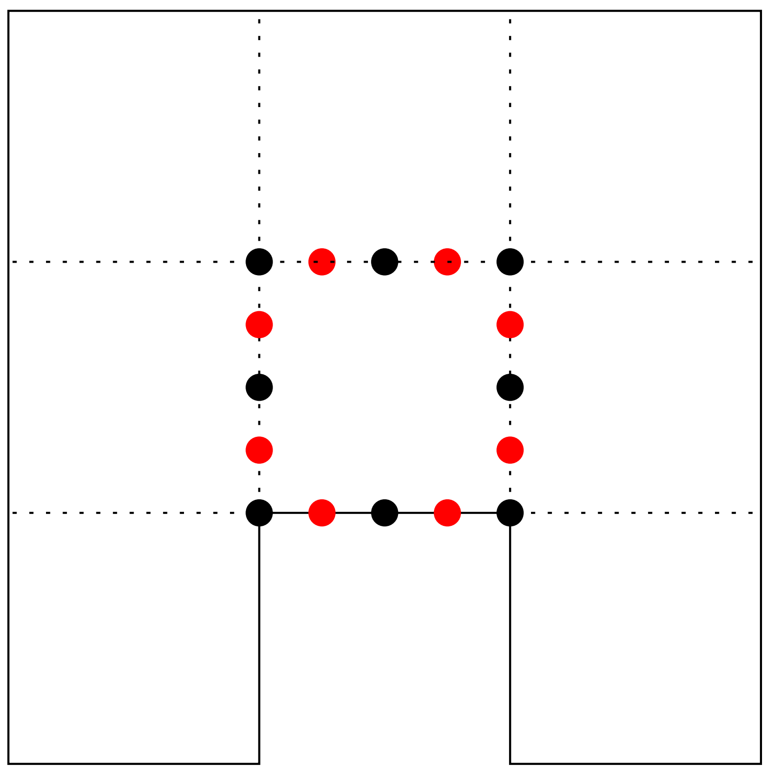
number of subdomains $N_{sd} = 8$

Setting up SLE



number of subdomains $N_{sd} = 8$
function nodes per subdomain $N_u^{sd} = 8$ ●

Setting up SLE

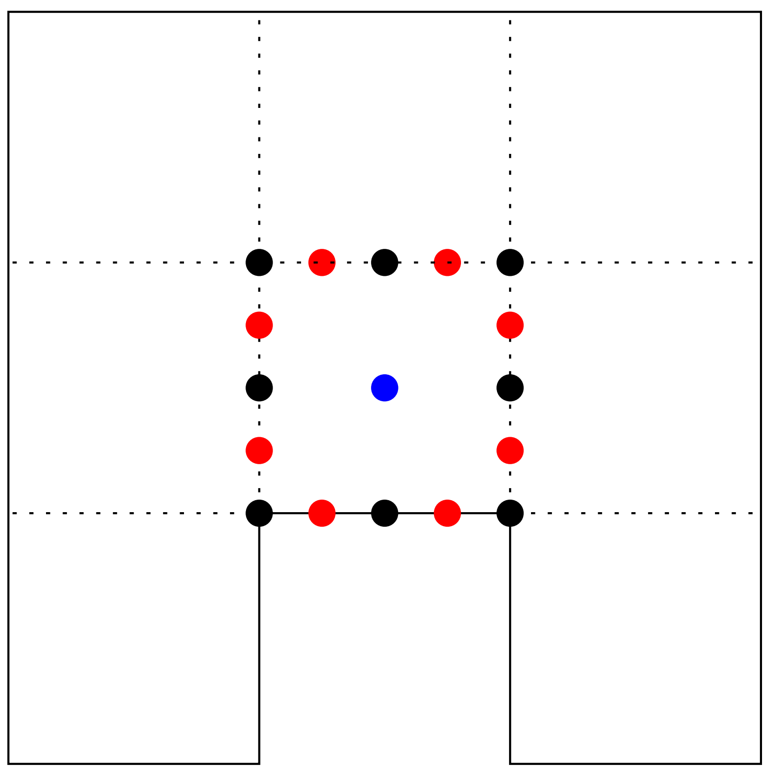


number of subdomains $N_{sd} = 8$

function nodes per subdomain $N_u^{sd} = 8$ ●

flux nodes per subdomain $N_q^{sd} = 8$ ●

Setting up SLE



number of subdomains $N_{sd} = 8$
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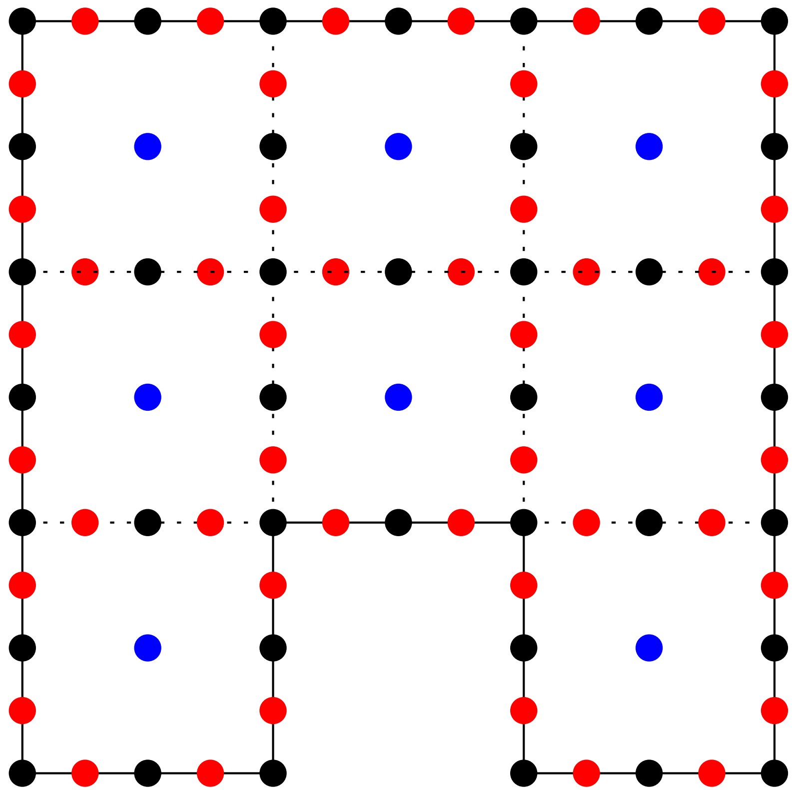
flux nodes per subdomain $N_q^{sd} = 8$
explicit nodes per subdomain $N_{ex}^{sd} = 1$

Setting up SLE



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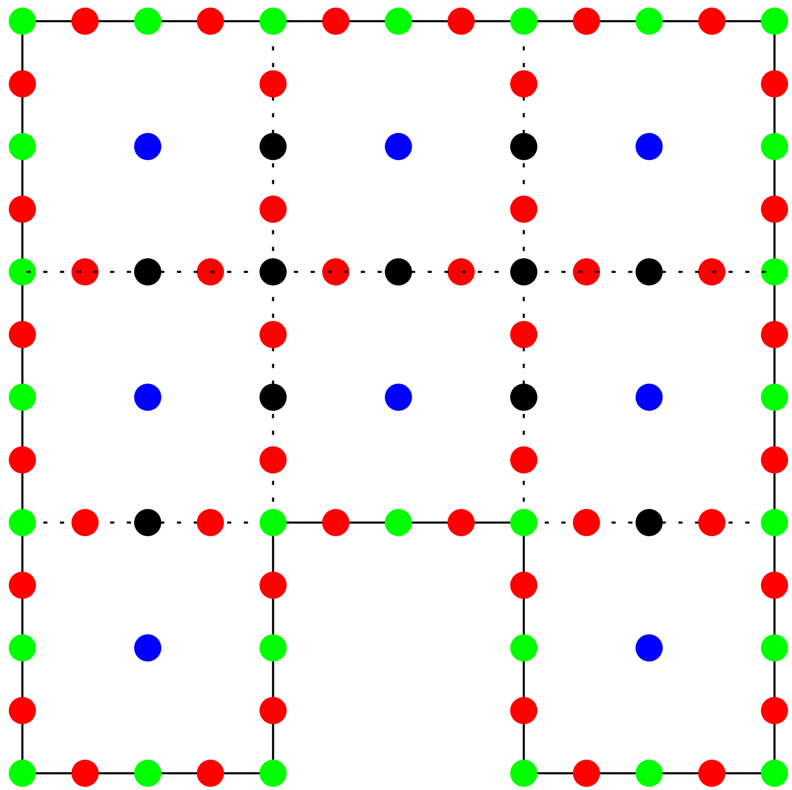
total number of function nodes $N_u = 38$
total number of flux nodes $N_q = 46$
total number of explicit nodes $N_{ex} = 8$

Setting up SLE



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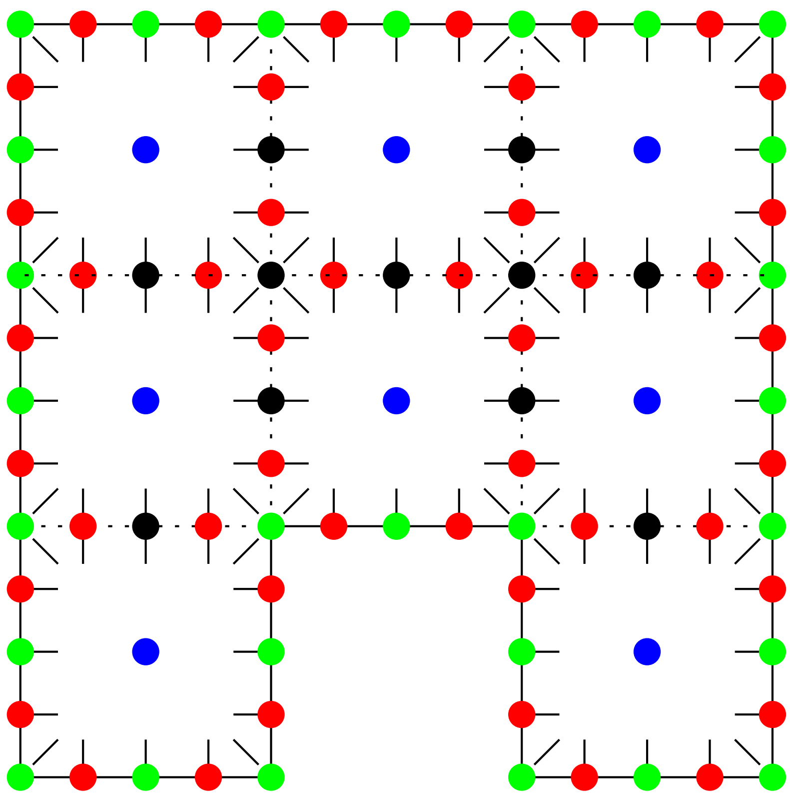
Dirichlet B.C. $N_{Dir} = 28$
number of unknowns $N_x = N_u + N_q - N_{Dir} = 56$

Setting up SLE



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Dirichlet B.C. $N_{Dir} = 28$
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number of equations
all source points $N_{eq} = N_{sd} \cdot (N_u^{sd} + N_q^{sd}) = 128$

Matrix approximation

- Single domain BEM is used to solve for boundary vorticity values.
- Resulting system of linear equations includes fully populated matrices.
- Domain integral yields matrices which scale as number of boundary nodes times number of all nodes.
- We approximate the domain matrix in the single domain solution of the kinematics equation by kernel expansion technique.

Poisson and kinematics eq.

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- The scalar Poisson and the vector kinematics equations

$$\nabla^2 u(\vec{r}) = b(\vec{r}); \qquad \nabla^2 \vec{v} + \vec{\nabla} \times \vec{\omega} = 0; \qquad \vec{r} \in \Omega,$$

- Their integral forms include a domain term

$$c(\vec{\theta})u(\vec{\theta}) + \int_\Gamma u(\vec{r})\vec{n} \cdot \vec{\nabla} u^\star d\Gamma = \int_\Gamma u^\star \vec{\nabla} u(\vec{r}) \cdot \vec{n} d\Gamma - \int_\Omega b(\vec{r}) u^\star d\Omega,$$

$$c(\vec{\theta})\vec{v}(\vec{\theta}) + \int_\Gamma \vec{v}\, \vec{\nabla} u^\star \cdot \vec{n} d\Gamma = \int_\Gamma \vec{v} \times (\vec{n} \times \vec{\nabla}) u^\star d\Gamma + \int_\Omega (\vec{\omega} \times \vec{\nabla} u^\star) d\Omega,$$

where $\vec{\theta}$ is the collocation point on the boundary, $\vec{n}$ is the unit normal and

$$u^\star = 1/4\pi|\vec{r} - \vec{\theta}|$$

is the fundamental solution of the Laplace equation in 3D.

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Kernel expansion

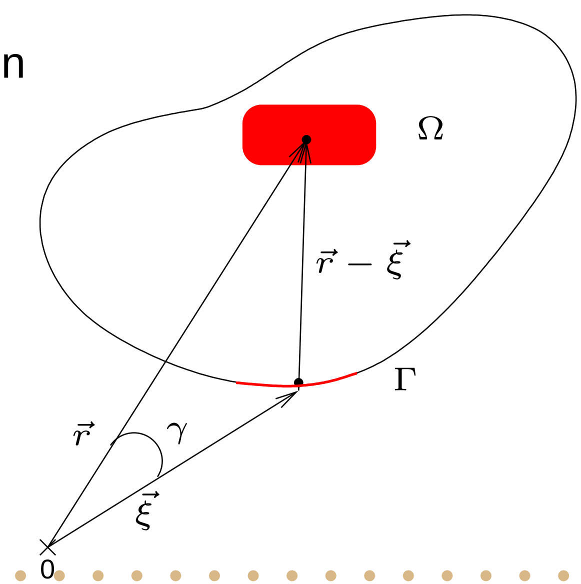
- The kernels of domain integrals are written in a spherical harmonics series to achieve separation of variables

$$\frac{1}{4\pi|\vec{r}-\vec{\theta}|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{(-1)^m}{2l+1} \frac{1}{\xi^{l+1}} Y_l^{-m}(\theta_\xi, \varphi_\xi) r^l Y_l^m(\theta_r, \varphi_r),$$

$$\vec{\nabla} \frac{1}{4\pi|\vec{r}-\vec{\theta}|} = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{(-1)^m}{2l+1} \frac{1}{\xi^{l+1}} Y_l^{-m}(\theta_\xi, \varphi_\xi) \left\{ l Y_l^m(\theta_r, \varphi_r) r^{l-2} \vec{r} + r^l \vec{\nabla} Y_l^m(\theta_r, \varphi_r) \right\},$$

where $\vec{r} = (r, \varphi_r, \theta_r)$ and $\vec{\theta} = (\xi, \varphi_\xi, \theta_\xi)$ are written in spherical coordinate system.

$$\frac{1}{4\pi|\vec{r}-\vec{\theta}|} = \sum_i F_i(\vec{\theta}) G_i(\vec{r})$$



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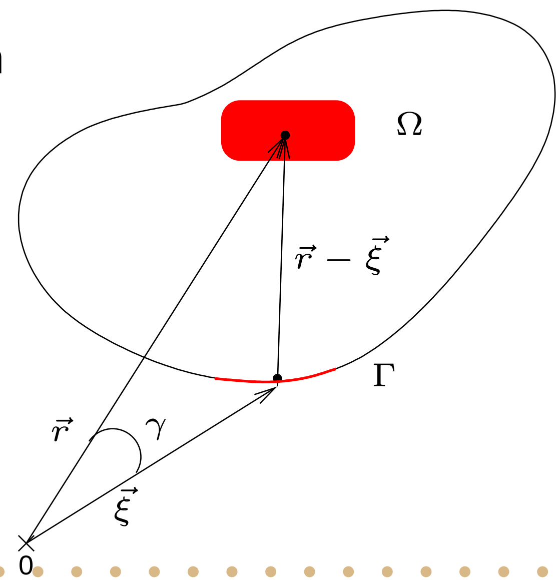

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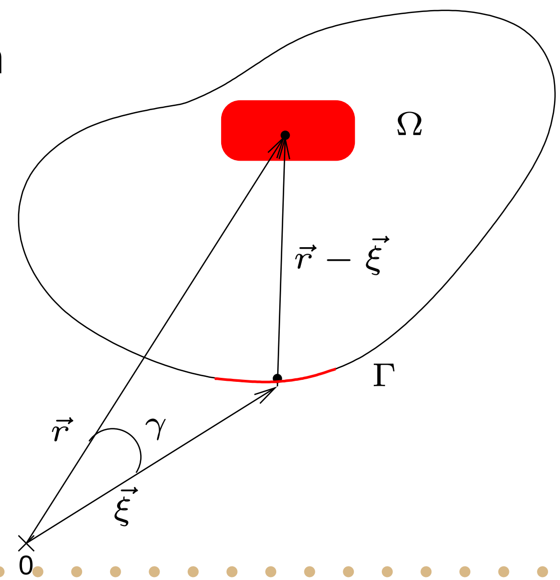

Writing $u^\star$ in a series

- A matrix block written as a product of two low rank matrices. n is the number of terms in the kernel expansion series, n_b is the number of collocation points and n_d is the number of domain points.

$$\begin{vmatrix} \int u^\star_{11} & \dots & \int u^\star_{1n_d} \\ \vdots & \ddots & \vdots \\ \int u^\star_{n_b 1} & \dots & \int u^\star_{n_b n_d} \end{vmatrix} \approx \begin{vmatrix} F_1^1 & F_2^1 & F_3^1 & \dots & F_n^1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ F_1^{n_b} & F_2^{n_b} & F_3^{n_b} & \dots & F_n^{n_b} \end{vmatrix} \cdot \begin{vmatrix} \int G_1^1 & \dots & \int G_1^{n_d} \\ \int G_2^1 & \dots & \int G_2^{n_d} \\ \int G_3^1 & \dots & \int G_3^{n_d} \\ \vdots & \ddots & \vdots \\ \int G_n^1 & \dots & \int G_n^{n_d} \end{vmatrix}$$

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- Compression is achieved if

$$n_d \cdot n_b > 2(n \cdot n_b + n \cdot n_d) = 2n(n_d + n_b),$$

where the factor 2 comes from the fact that F and G are complex values.

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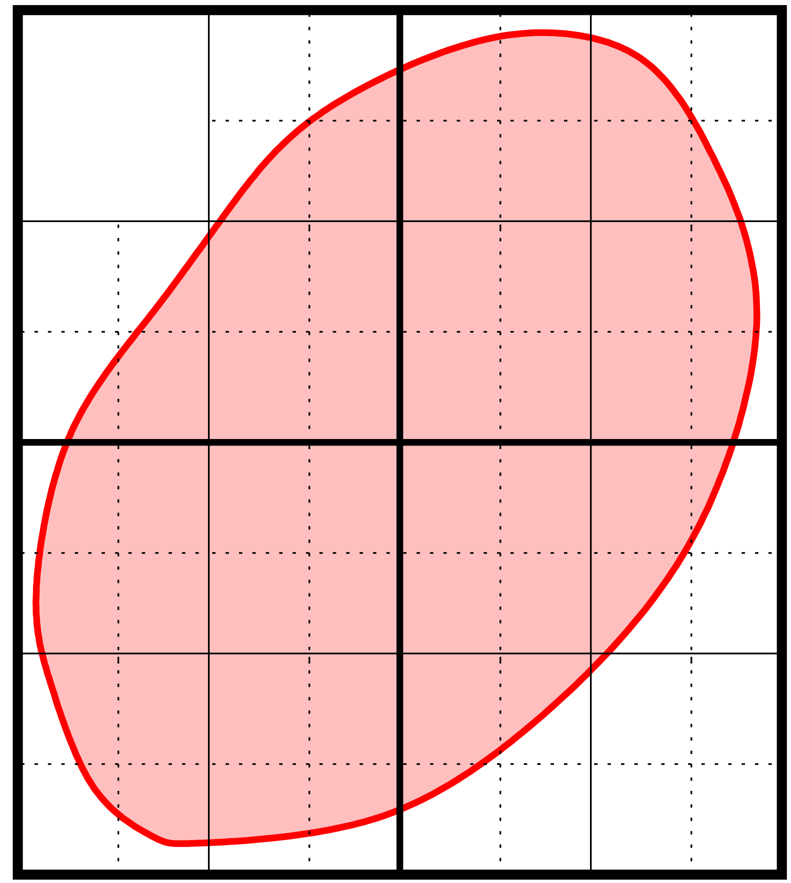
Hierarchical division of the domain

- The domain and boundary are cut up in a recursive hierarchical manner. Only a slice through a 3D domain is shown in the figure.
- Two trees of clusters are formed. The branches of the domain tree are clusters of domain elements. The branches of the boundary tree are clusters of boundary collocation nodes.
- Finally, by combining branches of both trees, a boundary-domain tree is formed, whose branches combine clusters of domain elements with clusters of boundary nodes.



Matrix structure - 33^3 nodes

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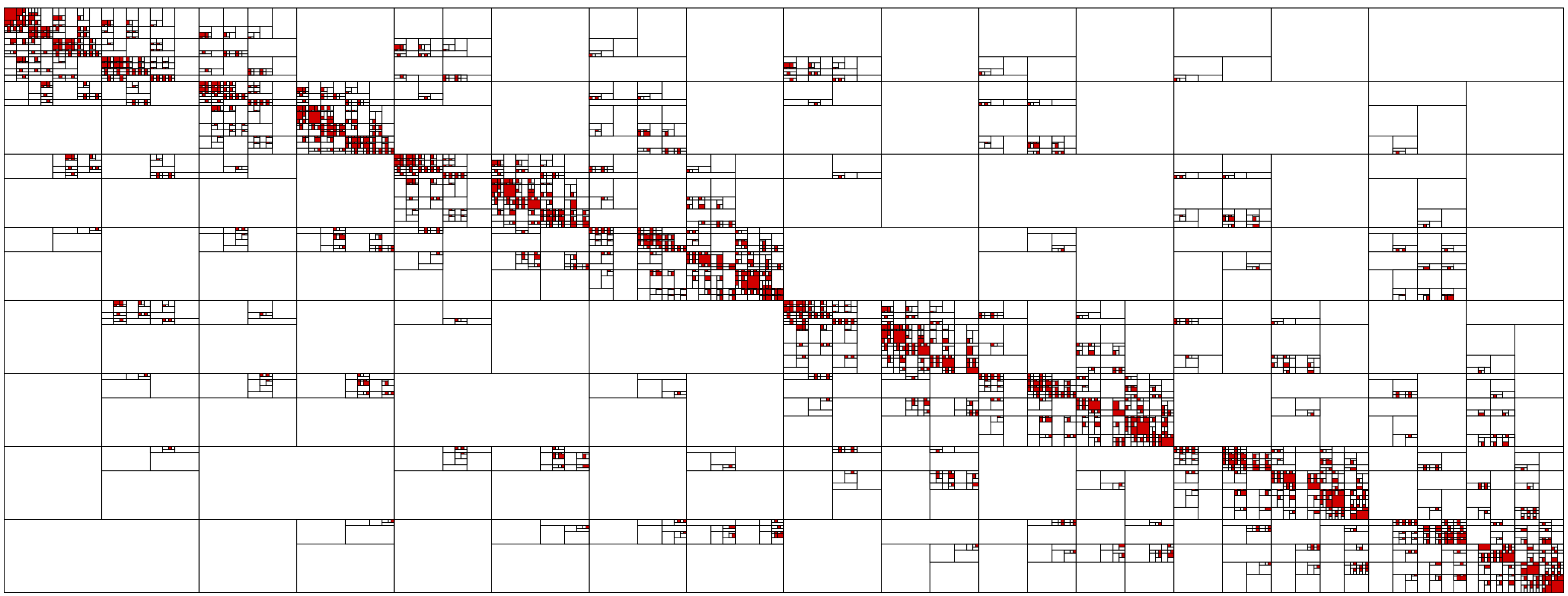
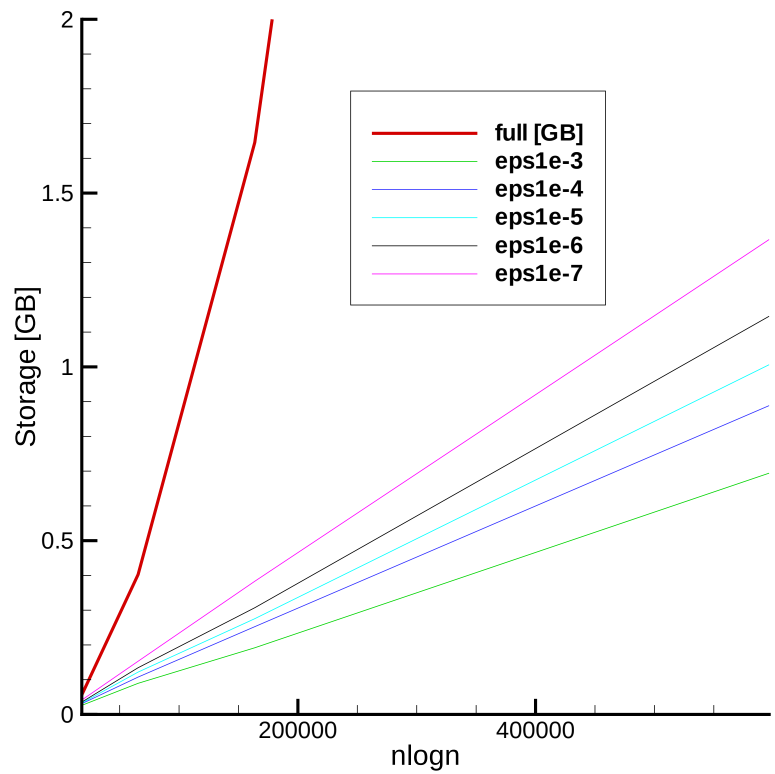
- Matrix structure of a cubic mesh with 33^3 nodes. Red areas show inadmissible parts of the matrix (leafs), white areas are admissible leafs obtained using an admissibility criteria of 10^{-5}. The corresponding boundary-domain tree had 19 levels. The compression ratio of this matrix representation is 0.167.



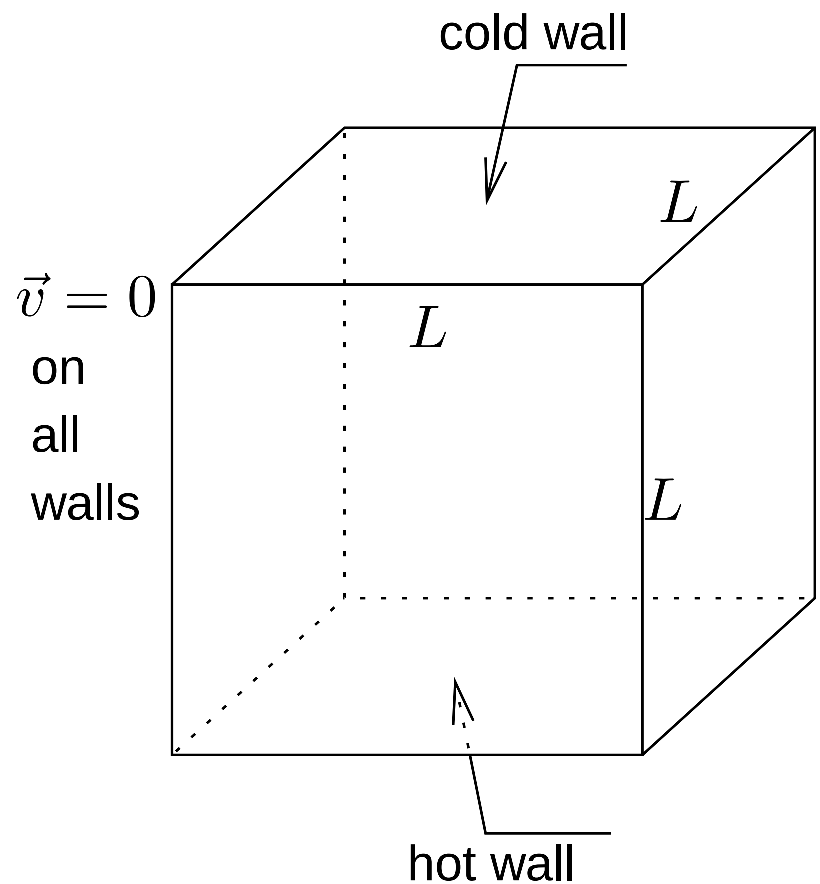
$\mathcal{O}(n \log n)$

- Comparison of storage requirements of a full matrix and a compressed matrix. It can clearly be seen that storage of the compressed matrix grows linearly with the number of nodes times the base 10 logarithm of n, making our algorithm $\mathcal{O}(n \log n)$. This relationship remains the same for a wide range of admissibility criteria (10^{-3} to 10^{-7}).



Rayleigh-Bénard convection

- The domain was a cubic cavity, where
  - the bottom wall was heated to a constant temperature and
  - the top wall cooled to an also constant but lower temperature.
  - the vertical walls of the cavity are insulated
  - no-slip velocity boundary conditions are applied on all walls
- The temperature difference between the walls defines the Rayleigh number for this case.



Rayleigh-Bénard convection

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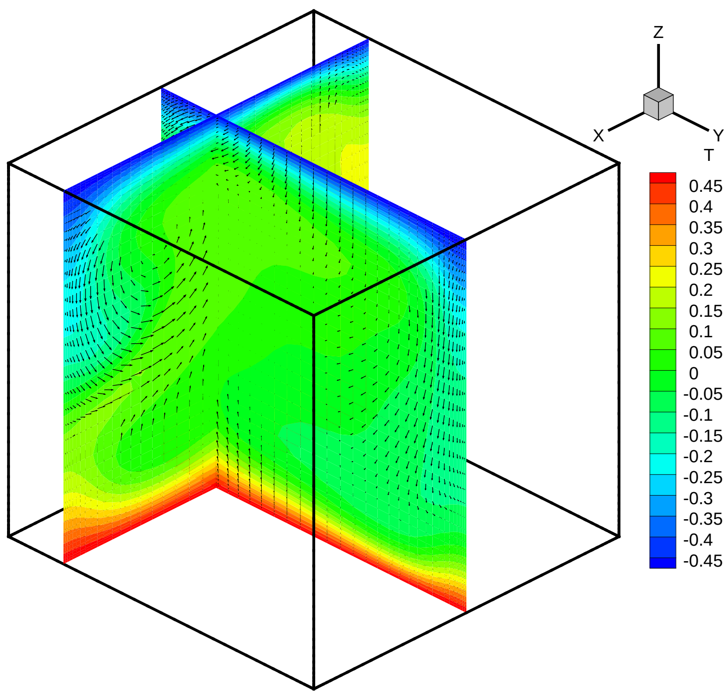
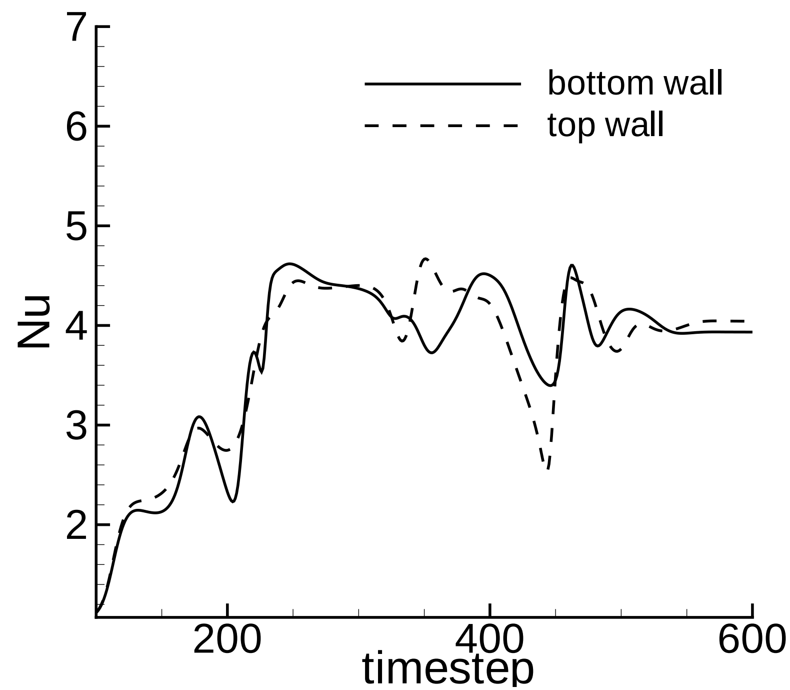
- Mesh of 20^3 elements with 41^3 nodes was used
- Simulation were run with air ($Pr = 0.71$) as the working fluid and Rayleigh number values of $Ra = 10^5$ and $Ra = 10^6$.
- Nondimensional time steps of $\Delta t = 10^{-3}$ and $\Delta t = 5 \cdot 10^{-4}$ were used.
- During the simulation heat transfer through the top and bottom walls was measured in terms of Nusselt number value.

Rayleigh-Bénard convection

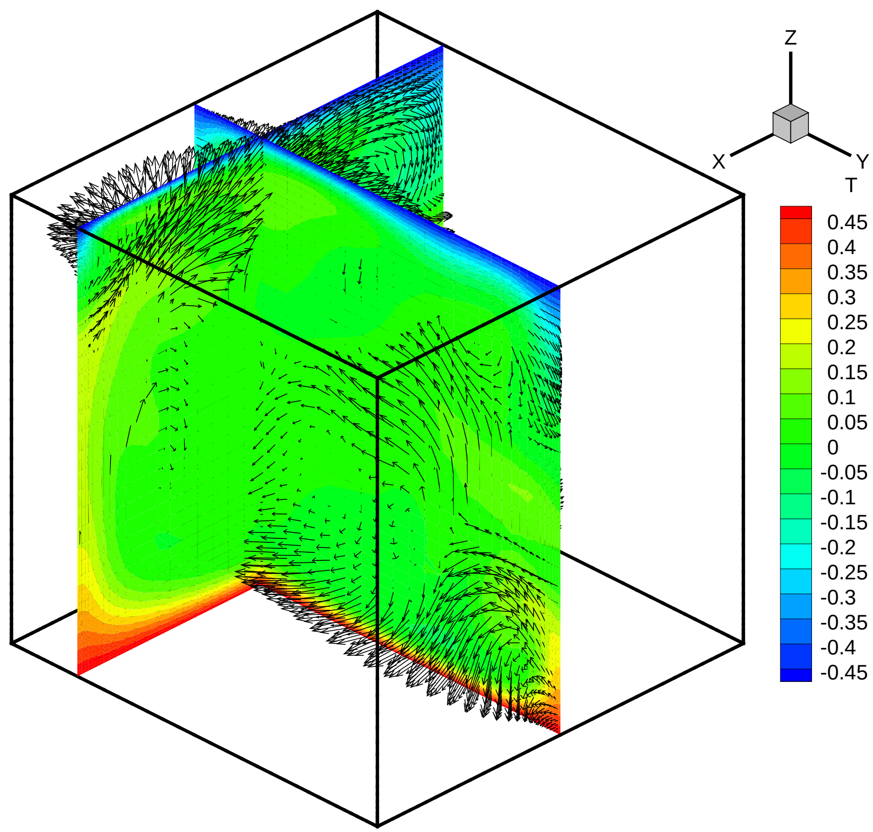
- At Rayleigh number $Ra = 10^5$ steady state was reached after 600 time steps.





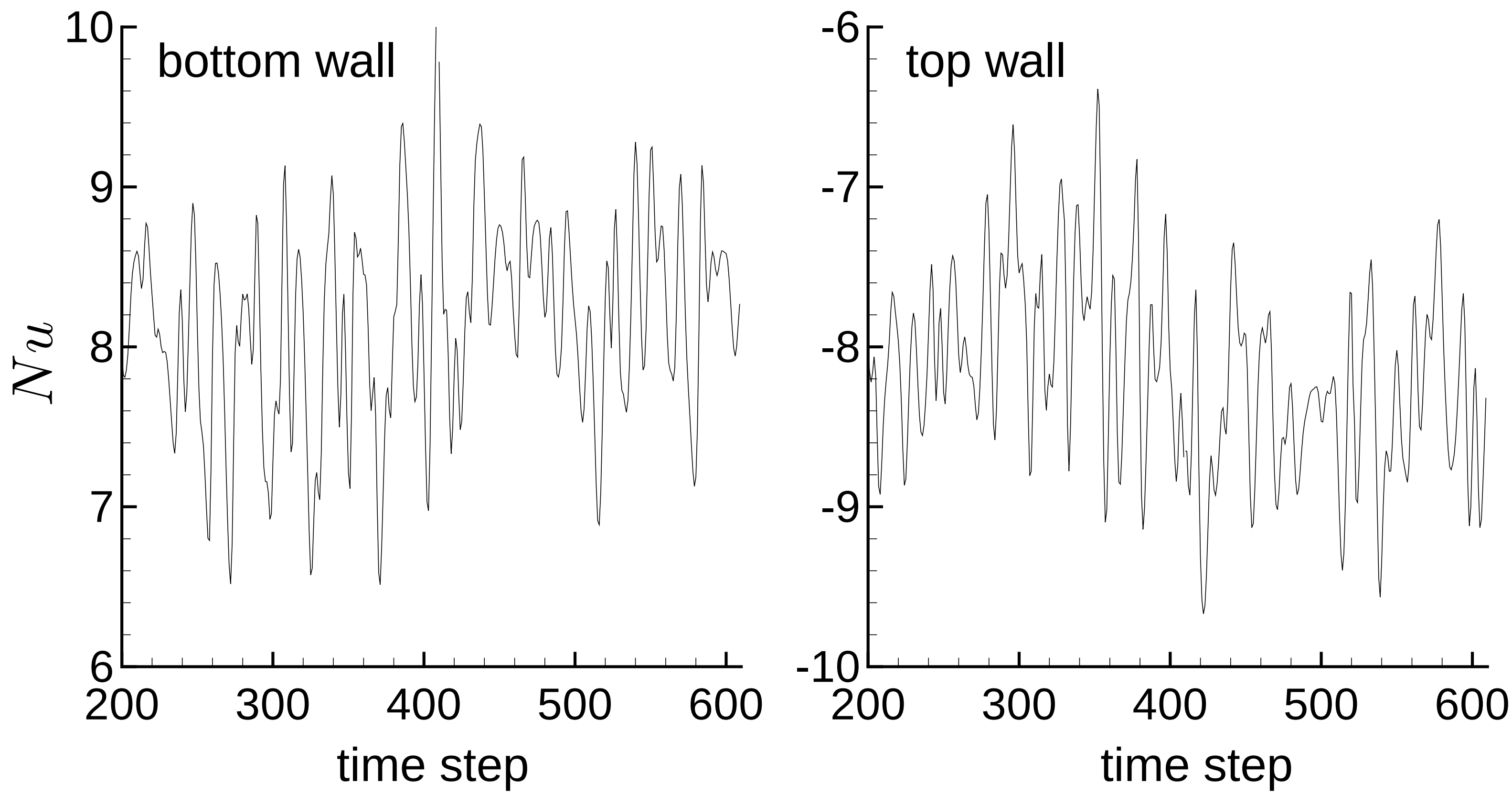
Rayleigh-Bénard convection

- At Rayleigh number $Ra = 10^6$ the flow is unsteady (movie).



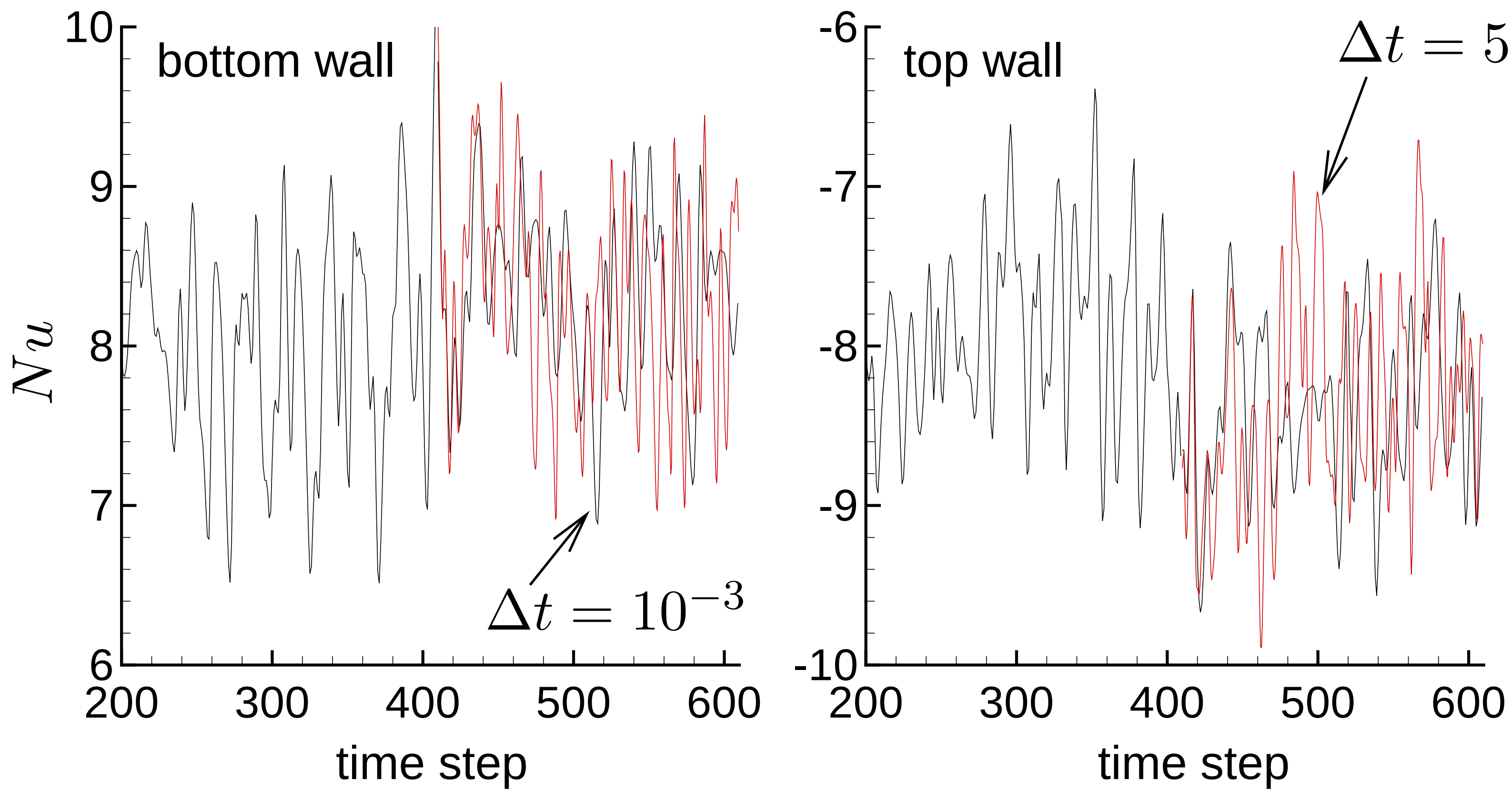
Nusselt number versus time

- $Ra = 10^6$



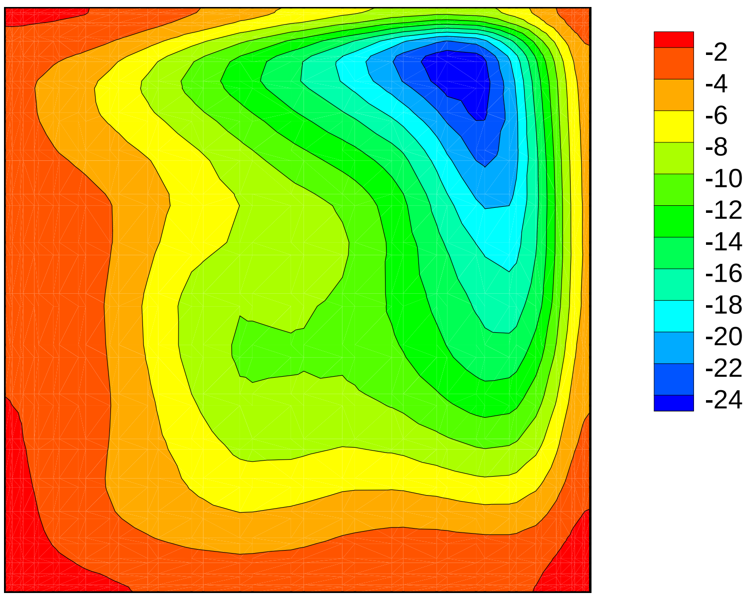
Nusselt number versus time

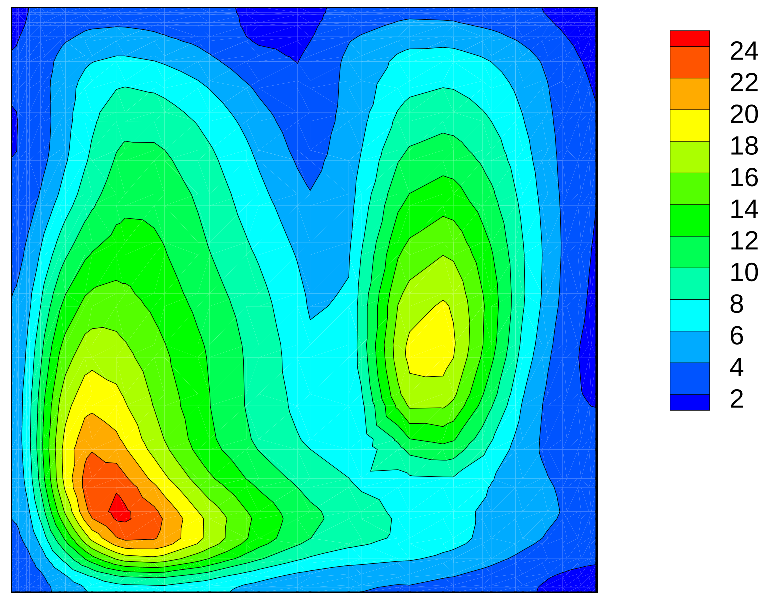
- $Ra = 10^6$



Rayleigh-Bénard convection

- A look at the heat flux (Nusselt number) through both walls (left: top, right: bottom) (movie).





Application - nanofluids

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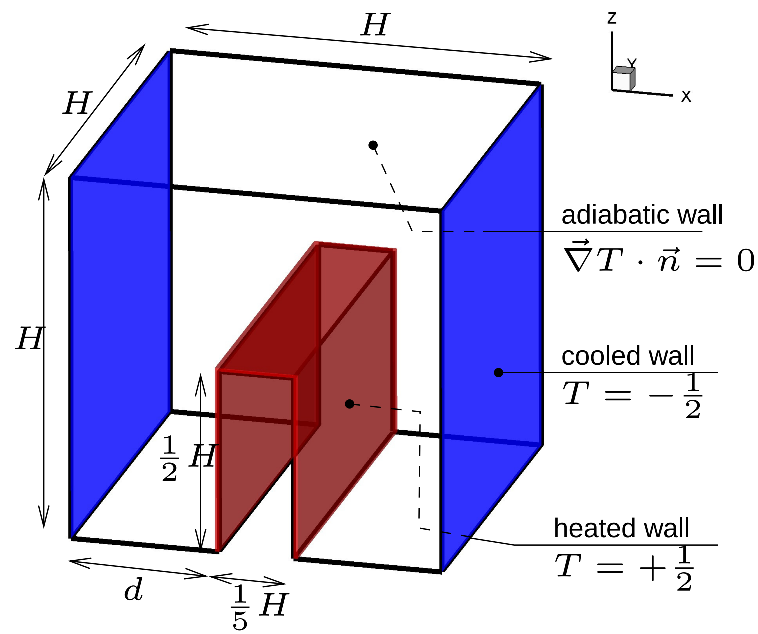
- The developed 3D BEM viscous flow solver was applied on a coupled flow and heat transfer problem.
- Natural convection of nanofluids is considered.
- Thermophysical properties of water based nanofluids.

	pure water	Cu	Al_2O_3	TiO_2
$c_p[J/kgK]$	4179	385	765	686.2
$\rho[kg/m^3]$	997.1	8933	3970	4250
$k[W/mK]$	0.613	400	40	8.9538
$\beta[\cdot 10^{-5} K^{-1}]$	21	1.67	0.85	0.9
$\alpha[\cdot 10^{-7} m^2/s]$	1.47	1163	131.7	30.7

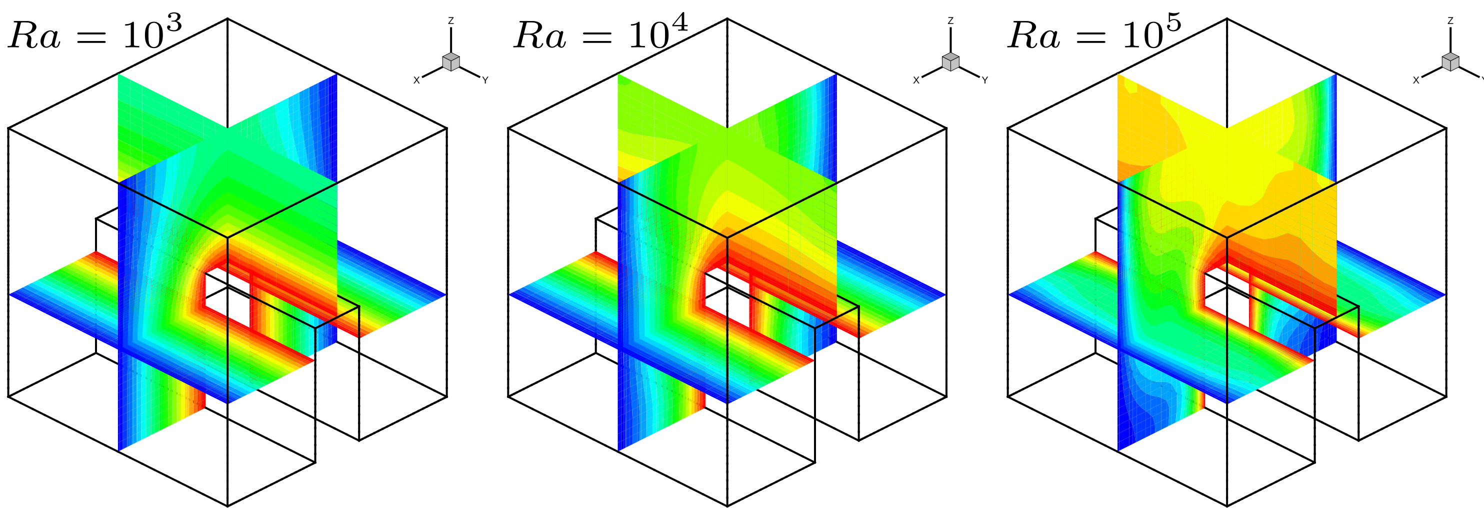
Nanofluids around a hotstrip

- Geometry of the hotstrip problem and boundary conditions.
- Two distances of the hotstrip from the left ($x = 0$) wall were considered; $d = 0.4H$ (central position) and $d = 0.5H$ and $d = 0.6H$.
- The width of the hotstrip is $0.2H$ in all cases. The hotstrip is heated to $T = +0.5$, while the walls as $x = 0$ and $x = H$ are cooled to $T = -0.5$. All other walls are adiabatic, i.e. there is no temperature flux through them.



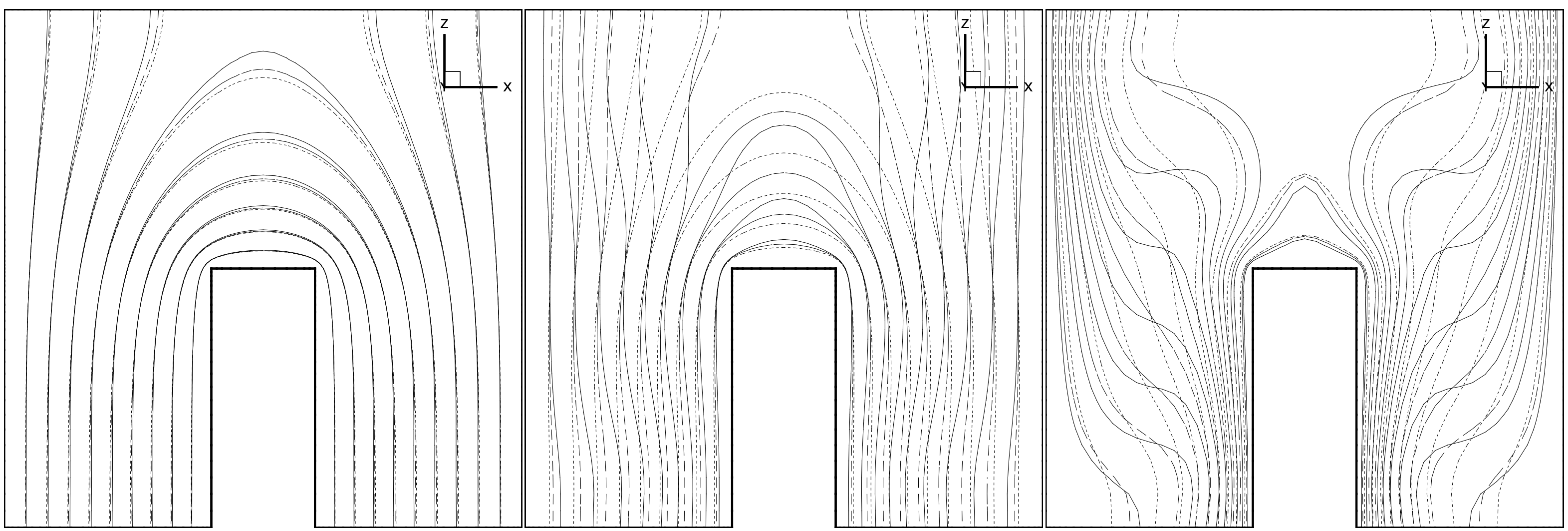
Flow description

- 3D simulation of natural convection around a hotstrip located in the central position ($d = 0.4H$).
- The hotstrip heats the surrounding air inducing two main vortices - one on each side of the hotstrip. At the top of the hotstrip, two smaller vortices are located. They keep the hot air close to the top of the hotstrip, making heat transfer from the top of the hotstrip small compared to the heat transfer from the sides of the hotstrip.
- Figures show temperature contours for central placement of the hotstrip.



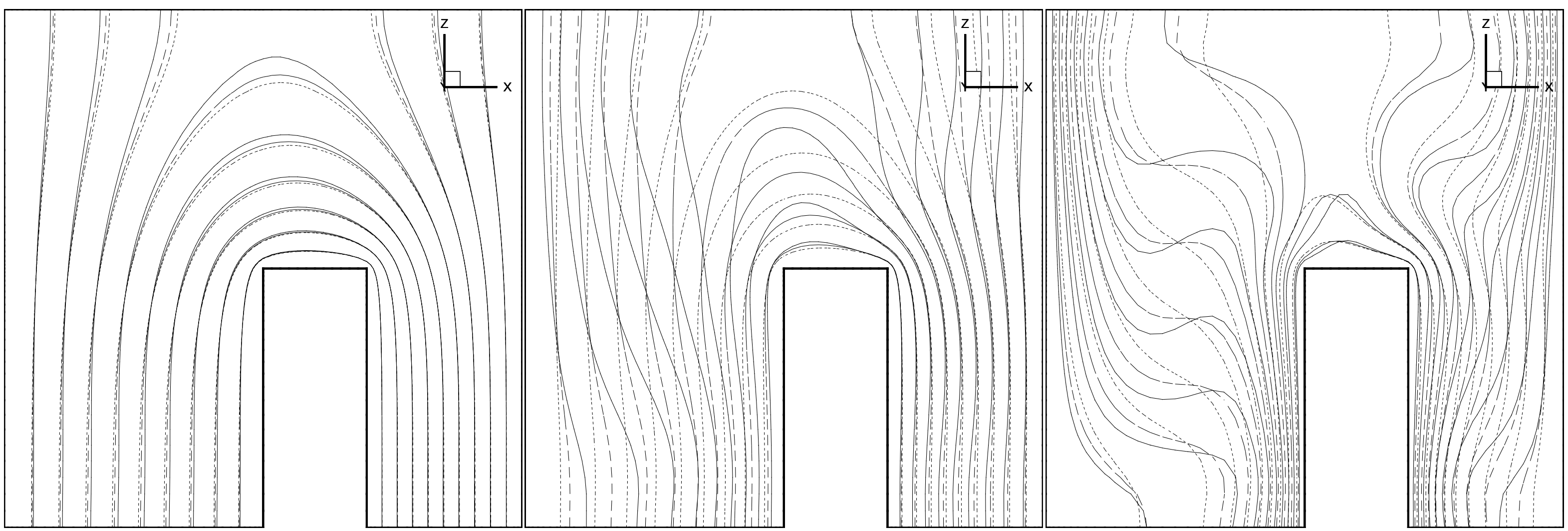
Temperature contours

- Temperature contours on the central $y = 0.5H$ plane for natural convection in a hotstrip located in the centre. Contour values are -0.4(0.1)0.4. Solid line denotes pure water, dashed line $\varphi = 0.1\ Cu$ nanofluid and dotted line $\varphi = 0.2\ Cu$ nanofluid. Left $Ra = 10^3$, middle $Ra = 10^4$ and right $Ra = 10^5$.



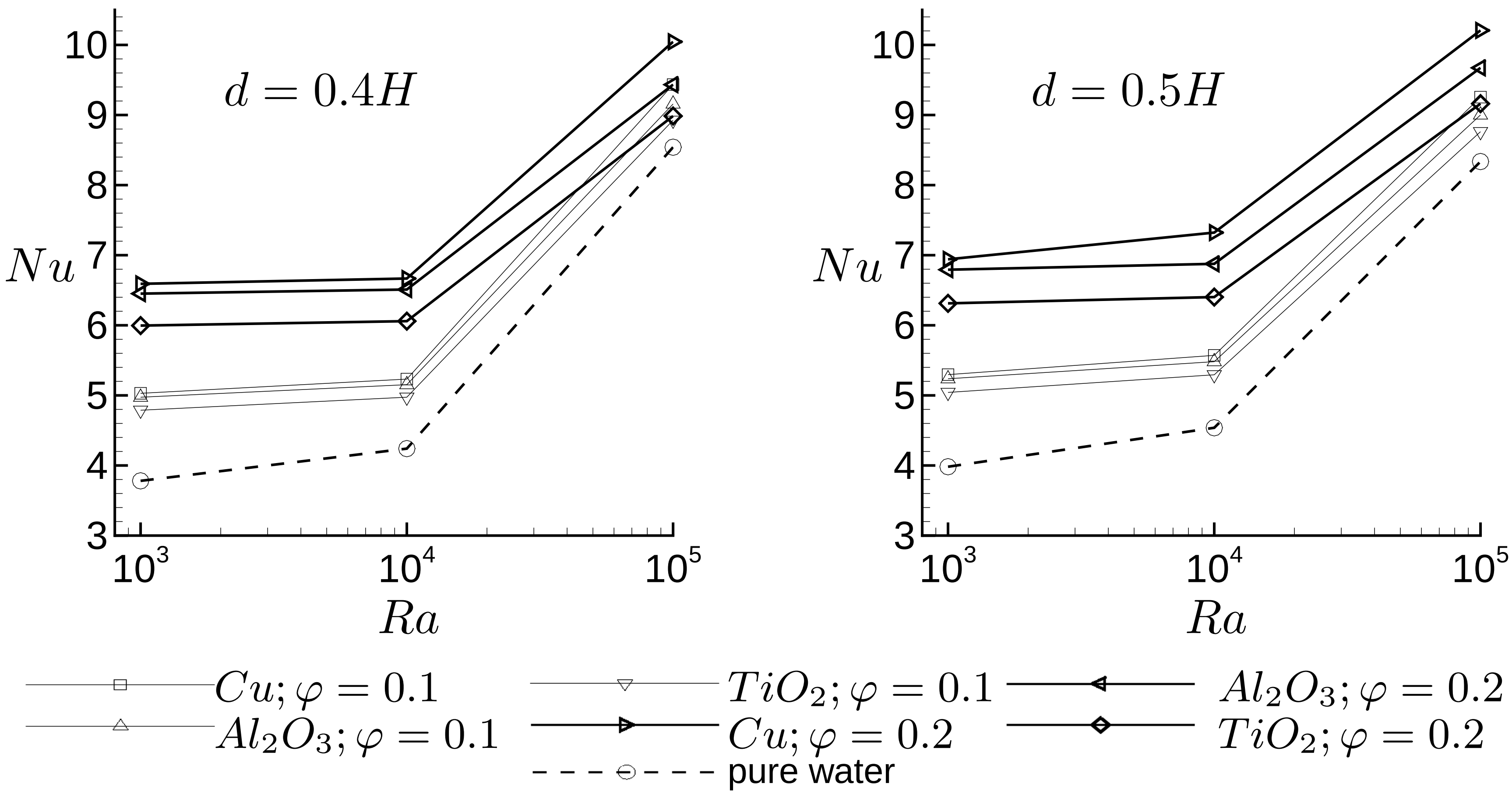
Temperature contours

- Temperature contours on the central $y = 0.5H$ plane for natural convection in a hotstrip located in the off-centre. Contour values are -0.4(0.1)0.4. Solid line denotes pure water, dashed line $\varphi = 0.1\ Cu$ nanofluid and dotted line $\varphi = 0.2\ Cu$ nanofluid. Left $Ra = 10^3$, middle $Ra = 10^4$ and right $Ra = 10^5$.



Heat transfer

- Total heat flux transferred from the hotstrip into the fluid presented with Nusselt number. Hotstrip is located in the centre of the cavity (left) and off-centre (right).



Conclusions

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- We presented a 3D laminar viscous flow and heat transfer solver.
- A combination of single domain BEM and sub-domain BEM was used to solve the velocity-vorticity formulation of Navier-Stokes equations.
- Domain decomposition makes BEM applicable to 3D viscous flows.
- Application of matrix approximation techniques depends on the nonlinearity of the problem.
- The algorithm was successfully used to study
  - Rayleigh-Bénard convection and
  - natural convection of nanofluids in a hotstrip heated enclosure. Steady and unsteady natural convection was considered.